ON SURFACE WAVES IN JOINED ELASTIC MATERIALS AND THEIR RELATION TO CRACK PROPAGATION ALONG THE JUNCTION

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Consider an elastic body consisting of two materials with different elastic properties. The junction is a horizontal line along which the contact conditions do not vary. The following are among the many possible contact conditions: 1) Complete adhesion (continuity of normal and shear stresses as well as vertical and horizontal displacements); 2) frictionless contact (continuity of normal stresses and vertical displacements with vanishing shear stresses); 3) nonslip with possible separation (continuity of shear stresses and horizontal displacements and the absence of normal stresses).

It is well known that in the case of complete adhesion of joined materials it is generally possible for Stoneley [1] surface waves to propagate along the junction line. The problem concerning the existence of these waves for joined materials with arbitrary properties has been investigated in [2 and 3].

The present investigation (Section 1) deals with surface waves which propagate along the boundary of joined materials in cases involving contact and nonslip with possible separation.

Just as in the case of Stoneley waves, the surface waves for bodies in contact do not exist for arbitrary relations between properties of the elastic materials, whereas for nonslip conditions between bodies, surface waves always exist. The velocity of these as well as other surface waves is bounded by the smaller of the Rayleigh velocities and the smaller of the velocities of sound in the two joined materials.

Note that papers [4, 5 and 6] and [6] contain prior studies of reflection and refraction of elastic waves along the junction line between two half-spaces for contacts of types (2) and (3), respectively.

The surface wave velocities in cases of contact and nonslip between bodies are the characteristic speeds obtained in problems of crack propagation along the junction. It was shown in [7] that for steady motion of a normal separation crack along the boundary junction, when the continuation of the crack consists of complete adhesion, there are, in addition to the speed of sound, five other characteristic speeds and that transition through these speeds leads to a change in the character of the stress distribution in the neighborhood of the crack tip. These speeds are the Rayleigh surface wave velocities in each of the half-spaces; the Stoneley wave velocities, and finally two velocities which, as will be seen, coincide with c_e and c_g, the surface wave velocities in cases of contact and nonslip respectively.

In this connection, the investigation in Section 2 deals with the steady motion of a "semi-infinite" crack with normal separation along a boundary with complete adhesion between materials subjected to concentrated shearing and normal loads, the crack velocity c coinciding with surface wave velocities obtained under contact and nonslip conditions.

It turns out that, under the action of normal loads as well as shear loads only, if $c = c_e$ or $c = c_g$ no free crack segment preceding the load exists.

An interesting property of the velocities c_e and c_g should be noted. If two free elastic half-spaces are deformed under identical normal surface loads moving with velocity c_e then the boundary displacements are such that these half-spaces may be fitted one against the other satisfying contact conditions along the boundary. A similar situation exists when identical shearing loads, moving with a velocity c_g, act on the boundary of free half-spaces The two half-spaces can be brought together so that the nonslip boundary conditions are satisfied.

1. 1.1°. Consider an elastic body consisting of two half-spaces which are joined along the x axis and which possess different elastic properties. All quantities associated with the upper ($y \geqslant 0$) and lower ($y \leqslant 0$) half-spaces will be designated by the subscripts 1 and 2, respectively. The following conditions are assumed to hold along the line of contact:

$$\sigma_{y1} = \sigma_{y2}, \quad v_1 = v_2, \quad \tau_{xy1} = \tau_{xy2} = 0 \quad (y = 0, \ -\infty < x < \infty) \tag{1.1}$$

Here σ_y and τ_{xy} are the normal and shear stresses, while v denotes vertical displacements of points on the boundary surface.

Thus, horizontal displacements may be discontinuous along the junction line, i.e. the half-spaces may slide against each other without friction. Such a joining may exist, for example, when elastic bodies are placed one on top of the other, and there is no friction along the line of contact.

The surface waves of an elastic body consisting of two half-spaces in contact will be sought in the exact same manner as was done in [2] in connection with the condition of complete adhesion ($\sigma_{y1} = \sigma_{y2}$, $\tau_{xy1} = \tau_{xy2}$, $u_1 = u_2$, $v_1 = v_2$) at the boundary, utilizing the procedure of V.I. Smirnov and Sobolev [8], applying complex variable methods to the wave equation.

We introduce the scalar and vector potentials $\varphi_{1,2}$ and $\psi_{1,2}$, respectively, satisfying the wave Eqs.

$$a_i^2 \Delta\varphi_i = \varphi_{itt}, \quad b_i^2 \Delta\psi_i = \psi_{itt}, \quad a_i^2 = (\lambda_i + 2\mu_i)/\rho_i, \quad b_i^2 = \mu_i/\rho_i \quad (i = 1, 2) \tag{1.2}$$

The contact boundary conditions (1.1) may then be written as:

$$\frac{\partial\varphi_1}{\partial y} - \frac{\partial\psi_1}{\partial x} - \left[\frac{\partial\varphi_2}{\partial y} - \frac{\partial\psi_2}{\partial x}\right] = 0$$

$$\mu_1\left[\left(\frac{a_1^2}{b_1^2} - 2\right)\Delta\varphi_1 + 2\frac{\partial^2\varphi_1}{\partial y^2} - 2\frac{\partial^2\psi_1}{\partial x\,\partial y}\right] - \mu_2\left[\left(\frac{a_2^2}{b_2^2} - 2\right)\Delta\varphi_2 + 2\frac{\partial^2\varphi_2}{\partial y^2} - 2\frac{\partial^2\psi_2}{\partial x\,\partial y}\right] = 0$$

$$2\frac{\partial^2\varphi_1}{\partial x\,\partial y} + \frac{\partial^2\psi_1}{\partial y^2} - \frac{\partial^2\psi_1}{\partial x^2} = 0, \qquad 2\frac{\partial^2\varphi_2}{\partial x\,\partial y} + \frac{\partial^2\psi_2}{\partial y^2} - \frac{\partial^2\psi_2}{\partial x^2} = 0 \tag{1.3}$$

Following [2 and 8], we will seek the potentials φ_i and ψ_i in the form

$$\varphi_1 = N_1 f(pt + \alpha x + iy\omega_{a1}), \qquad \psi_1 = N_2 f(pt + \alpha x + iy\omega_{b1})$$

$$\varphi_2 = N_3 f(pt + \alpha x - iy\omega_{a2}), \qquad \psi_2 = N_4 f(pt + \alpha x - iy\omega_{b2})$$

$$\omega_{aj} = (\alpha^2 - p^2/a_j^2)^{1/2}, \quad \omega_{bj} = (\alpha^2 - p^2/b_j^2)^{1/2} \quad (j = 1, 2) \tag{1.4}$$

Here $N_1, \ldots, N_4$ are constants while f is a function of a complex variable, regular in the upper half-space and possessing a derivative which vanishes at infinity. This guarantees the attenuation of oscillations as the distance from the boundary $y = 0$ increases, i.e. the oscillations will be surface waves. In addition, we will assume that the wave velocity does not surpass the smaller of the velocities of sound in the joined materials.

Substituting (1.4) into (1.3), we obtain a homogeneous system of linear equations in the four constants $N_1, \ldots, N_4$. For the existence of a nontrivial solution of this system, and consequently for the existence of natural oscillations of the joined bodies, it is necessary and sufficient that the following determinant vanish:

$$\begin{vmatrix} \mu_1\Omega_{b1} & -2\mu_1 i\alpha\omega_{b1} & -\mu_2\Omega_{b2} & -2\mu_2 i\alpha\omega_{b2} \\ 2i\alpha\omega_{a1} & \Omega_{b1} & 0 & 0 \\ 0 & 0 & 2i\alpha\omega_{a2} & -\Omega_{b2} \\ i\omega_{a1} & -\alpha & i\omega_{a2} & \alpha \end{vmatrix} = 0 \tag{1.5}$$

$$\Omega_{bj} = p^2/b_j^2 - 2\alpha^2 \qquad (j = 1, 2)$$

In expanded form, (1.5) takes the form

$$\mu_2 (p^2 / b_1^2)\, \omega_{a1} [\Omega_{b2}^2 - 4\alpha^2 \omega_{a2}\omega_{b2}] + \mu_1 (p^2 / b_2^2)\, \omega_{a2} [\Omega_{b1}^2 - 4\alpha^2 \omega_{a1}\omega_{b1}] = 0 \quad (1.6)$$

Taking into account the notation in (1.4) and (1.5) for ω and Ω, the above condition represents an equation for the determination of surface wave velocities in the joined bodies. The left-hand side of (1.6) coincides with the common denominator of the coefficients of reflection and refraction of elastic waves along the contact line between sliding elastic bodies, as determined in [5].

1.2°. Let $(p/|\alpha|) = c$, and rewrite (1.6) in the form

$$E(c) \equiv \mu_2 n_1^2 \sqrt{1-m_1^2}\, R_2(c) + \mu_1 n_2^2 \sqrt{1-m_2^2}\, R_1(c) = 0$$

$$R_j(c) = (2 - n_j^2)^2 - 4\sqrt{1-m_j^2}\sqrt{1-n_j^2}$$

$$m_j = c / a_j, \quad n_j = c / b_j \qquad (j = 1, 2) \quad (1.7)$$

Here $R_j(c)$ is the Rayleigh function for the jth half-space. The roots c_R^j of Eqs. $R_j(c) = 0$ are the velocities of the Rayleigh surface waves in each of the joined half-spaces.

We will now ascertain under what conditions (1.7) has roots when c lies in the interval 0 to b_1 (without restricting generality, it may be assumed that $b_1 < b_2$ and $c_{R1} < c_{R2}$). It may be shown (as it is done in [2] for the equation defining the velocity of Stoneley waves) that (1.7) cannot have more than one root in the interval $(0, b_1)$. Furthermore, it is easily seen that (1.7) has no roots for $0 \leqslant c \leqslant c_{R1}$. Indeed, $R_1 \leqslant 0$ for $c \leqslant c_{R1}$ and $R_2 \leqslant 0$ for $c \leqslant c_{R2}$, so that $E(c) < 0$ when $c \leqslant c_{R1}$. Thus, if the root c_* exists, it must lie between c_{R1} and b_1. A necessary and sufficient condition for the existence of a root of (1.7) defining the surface wave velocities in the joined bodies is given by

$$E(b_1) > 0 \quad (1.8)$$

The above condition is satisfied, for example, when the elastic properties of the joined materials do not differ very much, and the following relationship exists between the velocities of surface waves and transverse waves in the two half-spaces: $c_{R1} < c_{R2} < b_1 < b_2$. In that case, $E(b_1) > 0$, since $R_2 > 0$ for $c > c_{R2}$ and $R_1 > 0$ for $c > c_{R1}$. Moreover, under these circumstances, c_* lies between the Rayleigh velocities c_{R1} and c_{R2}, since $E(c_{R1}) < 0$ while $E(c_{R2}) > 0$. In particular, if an elastic body consists of two joined half-spaces both of which are of the same material, then $c_{R1} = c_{R2}$ and c_* simply coincides with the Rayleigh wave velocity c_R for this material. On the other hand, if the properties of the two materials differ sharply, for example, one of them, say the second, is absolutely rigid, then $E(b_1) < 0$, and (1.7) has no roots. This means that, in case of contact between an elastic body with a perfectly rigid body, no surface waves can propagate along the boundary.

1.3°. We will now seek surface waves in joined materials under conditions different from those of 1.1° and 1.2°. Suppose that along the junction line

$$\tau_{xy1} = \tau_{xy2}, \quad u_1 = u_2, \quad \sigma_{y1} = \sigma_{y2} = 0 \qquad (y = 0, \ -\infty < x < \infty) \quad (1.9)$$

Under these conditions, the materials may separate from each other, but they can not slide.

This may be described in the following manner. Assume that in each of the half-spaces there exists a series of small openings which are open to an perpendicular to the boundary. Assume further that the materials are placed against each other so that the openings are opposite each other and that in each pair of juxtaposed openings there is a frictionless thin peg. In such a junction, the pegs prevent sliding between the two bodies, but separation is not prevented.

In terms of the potential functions, the contact conditions take the form:

$$\mu_1\left[2\frac{\partial^2\varphi_1}{\partial x\,\partial y} + \frac{\partial^2\psi_1}{\partial y^2} - \frac{\partial^2\psi_1}{\partial x^2}\right] - \mu_2\left[2\frac{\partial^2\varphi_2}{\partial x\,\partial y} + \frac{\partial^2\psi_2}{\partial y^2} - \frac{\partial^2\psi_2}{\partial x^2}\right] = 0$$

$$\frac{\partial\varphi_1}{\partial x} + \frac{\partial\psi_1}{\partial y} - \left[\frac{\partial\varphi_2}{\partial x} + \frac{\partial\psi_2}{\partial y}\right] = 0 \quad (1.10)$$

$$\left(\frac{a_1^2}{b_1^2} - 2\right)\Delta\varphi_1 + 2\frac{\partial^2\varphi_1}{\partial y^2} - 2\frac{\partial^2\psi_1}{\partial x\,\partial y} = 0, \qquad \left(\frac{a_2^2}{b_2^2} - 2\right)\Delta\varphi_2 + 2\frac{\partial^2\varphi_2}{\partial y^2} - 2\frac{\partial^2\psi_2}{\partial x\,\partial y} = 0$$

If we again seek potential functions in the form (1.4), then the necessary and sufficient condition for the existence of a nontrivial solution takes the form

$$\begin{vmatrix} \mu_1 2i\alpha\omega_{a1} & \mu_1\Omega_{b1} & \mu_2 2i\alpha\omega_{a2} & -\mu_2\Omega_{b2} \\ \alpha & i\omega_{b1} & -\alpha & i\omega_{b2} \\ \Omega_{b1} & -2i\alpha\omega_{b1} & 0 & 0 \\ 0 & 0 & \Omega_{b2} & 2i\alpha\omega_{b2} \end{vmatrix} = 0 \qquad (1.11)$$

or, upon expansion of the determinant and taking into account (1.4) and (1.5), followed by the substitution ($[p/|\alpha|] = c$),

$$G(c) \equiv \mu_2 n_1^2 \sqrt{1-n_1^2}\, R_2(c) + \mu_1 n_2^2 \sqrt{1-n_2^2}\, R_1(c) = 0 \qquad (1.12)$$

It is easily seen that (1.12) has a single root c_g in the interval $(0, b_1)$ if and only if

$$G(b_1) \geqslant 0 \qquad (1.13)$$

Since $G(b_1) = \mu_1 (b_1^2/b_2^2)\sqrt{1-(b_1^2/b_2^2)} \geqslant 0$, the above condition is satisfied everywhere.

Thus, for arbitrary relations between the elastic properties of two materials which are joined so as to provide a nonslip condition but which are able to separate along the boundary, surface waves may propagate with a velocity c_g as defined by (1.12).

2. The surface wave velocities c_e and c_g in bodies which are in contact or which have nonslip joints turn out to be the characteristic velocities in problems of crack propagation along the boundaries of various types of joints.

As an example, consider the problem of steady motion of a semi-infinite crack having normal separation and propagated along the boundary by either normal or shearing concentrated loads, the extension of the crack being a joint with complete adhesion.

It can be shown that the Fourier transforms of stresses and displacements for the upper and lower half-spaces are interrelated at the boundary by the relations

$$\begin{aligned} \alpha U_1^- &= -\alpha U_1^+ + iA_1\Sigma_{y1}^+ + B_1 T_{xy1}^+ + (iA_1\Sigma_{y1}^- + B_1 T_{xy1}^-) \\ \alpha V_1^- &= -\alpha V_1^+ + C_1\Sigma_{y1}^+ - iA_1 T_{xy1}^+ + (C_1\Sigma_{y1}^- - iA_1 T_{xy1}^-) \qquad (\alpha > 0) \\ \alpha U_2^- &= -\alpha U_2^+ + iA_2\Sigma_{y2}^+ - B_2 T_{xy2}^+ + (iA_2\Sigma_{y2}^- - B_2 T_{xy2}^-) \\ \alpha V_2^- &= -\alpha V_2^+ - C_2\Sigma_{y2}^+ - iA_2 T_{xy2}^+ + (-C_2\Sigma_{y2}^- - iA_2 T_{xy2}^-) \end{aligned} \qquad (2.1)$$

$$\begin{aligned} \alpha U_1^- &= -\alpha U_1^+ + iA_1\Sigma_{y1}^+ - B_1 T_{xy1}^+ + (iA_1\Sigma_{y1}^- - B_1 T_{xy1}^-) \\ \alpha V_1^- &= -\alpha V_1^+ - C_1\Sigma_{y1}^+ - iA_1 T_{xy1}^+ + (-C_1\Sigma_{y1}^- - iA_1 T_{xy1}^-) \qquad (\alpha < 0) \\ \alpha U_2^- &= -\alpha U_2^+ + iA_2\Sigma_{y2}^+ + B_2 T_{xy2}^+ + (iA_2\Sigma_{y2}^- + B_2 T_{xy2}^-) \\ \alpha V_2^- &= -\alpha V_2^+ + C_2\Sigma_{y2}^+ - iA_2 T_{xy2}^+ + (C_2\Sigma_{y2}^- - iA_2 T_{xy2}^-) \end{aligned}$$

$$A_j = \frac{2\sqrt{1-m_j^2}\sqrt{1-n_j^2} - (2-n_j^2)}{\mu_j R_j}, \qquad B_j = \frac{n_j^2\sqrt{1-n_j^2}}{\mu_j R_j}, \qquad C_j = \frac{n_j^2\sqrt{1-m_j}}{\mu_j R_j} \qquad (j = 1, 2)$$

Here, c is the crack tip velocity.

The capital letters U, V, Σ and T denote the Fourier transforms of the corresponding quantities and α is the transform parameter.

The (+) and (−) designations denote Fourier transforms of functions which coincide with the desired solutions when $x \geqslant 0$ and $x \leqslant 0$, respectively, and which vanish for all other x. $U^\pm$, $V^\pm$, $\Sigma^\pm$ and $T^\pm$ are the values obtained in the limit, on the real axis, for the complex variable function $\zeta = \alpha + i\gamma$; they are analytic in the upper and lower halves of the ζ plane, respectively.

2.1°. Let us first examine the case of a crack propagated by concentrated forces which are normal to the boundary. The conditions along the line $y = 0$ are given by

$$\begin{aligned} &\sigma_{y1} = \sigma_{y2} = -Q\delta(x+l), \quad \tau_{xy1} = \tau_{xy2} = 0 \quad (x < 0) \\ &u_1 = u_2, \quad v_1 = v_2, \quad \tau_{xy1} = \tau_{xy2}, \quad \sigma_{y1} = \sigma_{y2} \quad (x > 0) \end{aligned} \qquad (2.2)$$

It has been shown [7] that, in this case, transition through the velocities c_e and c_g is associated with a change in the types of singularities obtained for the stresses at the crack tip. Let us examine what happens when the crack velocity c coincides with the surface wave velocity c_e of contacting bodies.

From (2.1), with the aid of a Fourier transform of the boundary conditions and (2.2), we obtain

$$\alpha(V_1^- - V_2^-) = (C_1 + C_2)\Sigma_{y1}^+ - i(A_1 - A_2)\mathrm{T}_{xy1}^+ + (G_1 - G_2)$$

$$\alpha(U_1^- - U_2^-) = i(A_1 - A_2)\Sigma_{y1}^+ + (B_1 + B_2)\mathrm{T}_{xy1}^+ + (F_1 - F_2)$$

$$\alpha V_2^- = -\alpha V_1^+ - C_2\Sigma_{y1}^+ - iA_2\mathrm{T}_{xy1}^+ + G_2, \quad \alpha U_2^- = -\alpha U_1^+ + iA_2\Sigma_{y1}^+ - B_2\mathrm{T}_{xy1}^+ + F_2$$

$$(\alpha > 0) \tag{2.3}$$

$$\alpha(V_1^- - V_2^-) = -(C_1 + C_2)\Sigma_{y1}^+ - i(A_1 - A_2)\mathrm{T}_{xy1}^+ - (G_1 - G_2)$$

$$\alpha(U_1^- - U_2^-) = i(A_1 - A_2)\Sigma_{y1}^+ - (B_1 + B_2)\mathrm{T}_{xy1}^+ + (F_1 - F_2)$$

$$\alpha V_2^- = -\alpha V_1^+ + C_2\Sigma_{y1}^+ - iA_2\mathrm{T}_{xy1}^+ - G_2, \quad \alpha U_2^- = -\alpha U_1^+ + iA_2\Sigma_{y1}^+ + B_2\mathrm{T}_{xy1}^+ + F_2$$

$$(\alpha < 0)$$

Here

$$G_1 = -QC_1e^{-i\alpha l}, \qquad G_2 = QC_2e^{-i\alpha l}, \qquad F_j = -iQA_je^{-i\alpha l} \qquad (j = 1, 2).$$

If the crack velocity $c = c_e$, then in view of (1.7) and (2.1),

$$C_1 + C_2 = E(c)/\mu_1\mu_2R_1R_2 = 0$$

and the first two relations in (2.3) take the form

$$\alpha(V_1 - V_2)^- = -i(A_1 - A_2)\mathrm{T}_{xy1}^+ \qquad (-\infty < \alpha < \infty) \tag{2.4}$$

The requirement of integrability of the stresses at the crack tip (the absence of concentrated forces at the crack tip) leads to the condition $\mathrm{T}_{xy1}^+ \to 0$ for $\alpha \to \infty$; hence it follows from (2.4) that $\mathrm{T}_{xy1}^+ = 0$ and $V_1^- - V_2^- = 0$. This means that, for $c = c_e$, there are no shear stresses $\tau_{xy} = 0$ $(x > 0)$, and the vertical displacements of the crack edges coincide $v_1 = v_2$ $(x < 0)$; i.e. the crack edges are in contact with each other. Now, from the second pair of relations (2.3) we obtain

$$\alpha(U_1 - U_2)^- = i(A_1 - A_2)\Sigma_{y1}^+ - iQ(A_1 - A_2)e^{-i\alpha l} \qquad (-\infty < \alpha < \infty) \tag{2.5}$$

It is clear from (2.5) that $i(A_1 - A_2)\Sigma_{y1}^+$ and $\alpha(U_1 - U_2)^-$ are equal to the limit values of the Cauchy type integral

$$W(\zeta) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{iQ(A_1 - A_2)e_2^{-itl}\,dt}{t - \zeta} \tag{2.6}$$

when $\zeta = \alpha + i\gamma$ approaches the real axis (path of integration) from above and below, respectively. It is easily seen that $W^+ = 0$, and, $W^- = i(A_1 - A_2)Q^{-i\alpha l}$. Thus, $\Sigma_{y1}^+ = 0$, and, since $U_1^+ - U_2^+ = 0$, $\alpha(U_1 - U_2) = -iQ(A_1 - A_2)e^{-i\alpha l}$. Performing the transformation and taking into account the condition of adhesion for the crack continuation, we obtain

$$u_1 - u_2 = -Q(A_1 - A_2) \;\text{ for }\; x < -l; \; u_1 - u_2 = 0 \;\text{ for }\; x > -l \tag{2.7}$$

Furthermore, $\sigma_{y1} = 0$ for $x > 0$, and this in conjunction with (2.2) implies that the normal stresses along the junction line differ from zero only at the points where the moving concentrated loads are located instantaneously.

Let us compute the vertical displacements for points along the junction line. Since $V_1^- = V_2^-$ and $\mathrm{T}_{xy1}^+ = \Sigma_{y1}^+ = 0$, the third pair of relations (2.3) may be written in the form

$$\alpha V_1 = QC_2e^{-i\alpha l} \;(\alpha > 0), \qquad \alpha V_1 = -QC_2e^{-i\alpha l} \quad (\alpha < 0) \tag{2.8}$$

Whence inversion yields

$$v_1(x, 0) = v_2(x, 0) = -\frac{QC_2}{\pi}\ln|x + l| + \text{const} \tag{2.9}$$

Thus, the distribution of the vertical displacements at boundary points is similar to that resulting from the action of a concentrated load on a half-space. A similar pattern exists for the horizontal displacements. As in the case of a normal load acting on a half-space, the horizontal displacements on the junction boundary in the first and second half-spaces to the right and left of the point of application of the load $(x = -l)$ take on constant values.

Moreover, as (2.7) indicates, for $x > -l$, u_1 and u_2 coincide (as a result of complete adhesion along the crack continuation), and for $x < -l$ the difference between horizontal displacements is constant.

Thus, if the velocity of the normal loads producing the crack coincides with the surface wave velocity in the contacting bodies c_e, then the conditions for complete adhesion are automatically satisfied at all points ahead of the load application point $x = -l$. No segment of free crack can exist ahead of the load (i.e. $l = 0$). Indeed, the energy necessary for separating the material must be absorbed at the tip of the propagating crack. If the crack speed is $c = c_e$ no stresses exist at the crack tip $x = 0$ (assuming that $l \neq 0$) and the energy flow at the tip equals zero. Thus, for $c = c_e$ the crack tip coincides with the point of application of the load.

The crack edges are in contact, and the stress and displacement distributions at the junction boundary for each of the two half-spaces is the same as in the case of a corresponding normal load moving along the free boundary of the corresponding half-space with velocity c_e. This means that for $c = c_e$ the normal separation crack ceases to exist. Tensile loads which are normal to the junction and moving with a velocity $c = c_e$ can not separate the two materials. The deformations of the joined half-spaces are such that they remain in contact. Moreover, as can be seen from (2.9), under a compressive load the first half-space will become convex while the second will become concave. The preceding phenomenon arises from the fact that the surface wave velocity in the first of the contacting materials is above the Rayleigh velocity ($c_e > c_{R1}$) while for the second it is below ($c_e < c_{R2}$). It is known [9 and 19] that, in problems of steadily moving loads and punches acting on the free boundary of a half-space, transition through the Rayleigh velocity produces a change in sign for the displacements and stresses.

Now let us assume that the crack tip velocity c equals the surface wave velocity c_g in nonslipping bodies. Thus, as may be seen from (1.12) and (2.1),

$$B_1 + B_2 = G(c) / \mu_1\mu_2 R_1(c) R_2(c) = 0$$

Hence, (2.3) yields in particular

$$\alpha (U_1 - U_2)^- = i (A_1 - A_2) \Sigma_{y1}^+ - iQ (A_1 - A_2) e^{-i\alpha l} \qquad (-\infty < \alpha < \infty) \tag{2.10}$$

Whence it follows that $\Sigma_{y1}^+ = 0$, i.e. the normal stresses on the crack continuation vanish, and that

$$\alpha (U_1 - U_2) = -i (A_1 - A_2) Q e^{-i\alpha}$$

i.e. the difference between horizontal displacements is constant

$$u_1 - u_2 = -Q (A_1 - A_2) \quad \text{for} \quad x < -l; \qquad u_1 = u_2 \quad \text{for} \quad x > -l$$

It can be shown that, in this case, the shearing stresses along the crack continuation are nonzero and possess no singularities at the crack tip

$$\tau_{xy1} = Q (C_1 + C_2) / \pi (A_1 - A_2)(x + l) \qquad \text{for} \quad x > 0$$

Furthermore, the difference between vertical displacements of the crack edges is given by

$$v_1 - v_2 = Q (C_1 + C_2) / \pi \ln |x + l| + \text{const}$$

Since the stresses in the neighborhood of the crack tip $x = 0$ are finite, assuming $l \neq 0$, then the energy flow at the crack tip is equal to zero. This means (just as in the case of $c = c_e$) that steady motion of a normal separation crack moving with velocity $c = c_g$ along the boundary can occur only if there is no free crack segment ahead of normal loads causing the crack ($l = 0$).

2.2°. Suppose that a normal separation crack is propagated by concentrated forces of magnitude Q and directed along the junction line. Suppose further that complete adhesion conditions exist along the crack continuation, whereupon, for $y = 0$

$$\begin{aligned} &\tau_{xy1} = \tau_{xy2} = -Q\delta(x + l), \qquad \sigma_{y1} = \sigma_{y2} = 0 \qquad (x < 0) \\ &\sigma_{y1} = \sigma_{y2}, \quad \tau_{xy1} = \tau_{xy2}, \quad u_1 = u_2, \quad v_1 = v_2 \quad (x > 0) \end{aligned} \tag{2.11}$$

The Fourier transforms for the displacements and stresses on the boundary are interrelated by relations similar to (2.3). If $c = c_g$, then, as in Subsection 2.1°, it can be shown that the normal and shear stresses along the crack continuation vanish $\sigma_{y1} = 0$, $\tau_{xy1} = 0$ $(x > 0)$; the horizontal displacements coincide along the entire junction line

$$u_1 = u_2 = -(QB_2 / \pi) \ln |x + l| + \text{const}$$

The vertical displacements are constant for each of the half-spaces both to the left and to the right of the point of application of the load, and their differences are given by

$$v_1 - v_2 = Q\,(A_1 - A_2) \quad \text{for} \quad x < -l, \qquad v_1 - v_2 = 0 \quad \text{for} \quad x > -l$$

If $c = c_e$, then the shear stresses along the crack continuation vanish $\tau_{xy1} = 0\,(x > 0)$; the normal stresses have no singularities at the crack tip

$$\sigma_{y1} = -\,Q\,(B_1 + B_2)\,/\,\pi\,(A_1 - A_2)\,(x + l) \quad \text{for} \quad x > 0$$

The difference between horizontal displacements of the crack edges equals

$$u_1 - u_2 = (QB_1\,/\,\pi)\,\ln|x + l| + \text{const} \quad (x < 0)$$

and the difference between vertical displacements is constant

$$v_1 - v_2 = Q\,(A_1 - A_2) \quad \text{for} \quad x < -l, \text{ и } v_1 - v_2 = 0 \quad \text{for} \quad x > -l$$

Thus, as in the case of cracks propagated by normal loads, steady motion of the crack under the action of shearing loads moving with velocities c_e and c_g is possible only when the load acts at the crack tip.

The role of the velocity c_g in the case of shearing loads is the same as that of the velocity c_e in the case of normal loads. If two free elastic half-spaces are subjected to the action of identical surface shearing loads moving with the velocity c_g, then the deformation of each will be such that they will remain in contact with each other, satisfying nonslip conditions along the junction boundary and complete adhesion conditions at all points ahead of the load.

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